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## Imperfect competition, technical progress and capital accumulation

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# Imperfect competition, technical progress and capital accumulation\*

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## Abstract

This paper explores the consequences of imperfect competition on capital accumulation. The framework is an OLG growth model with altruistic agents. Two types of long run equilibria exist: egoistic or altruistic. We assume both competitive and non-competitive firms exist, the latter being endowed with more productive technology. They behave strategically on the labor market: they take into account the impact of their demand for labor on the equilibrium wage and on their profit. The effect of technical progress for a non-competitive firm depends on the initial productivity of the firm and on the type of steady state (egoistic or altruistic). An increase in the productivity of the most productive firm has a negative impact on capital accumulation in an egoistic steady state, and a positive one in an altruistic steady state. An increase in the productivity of the competitive sector can have various effects on capital accumulation. If the productivity levels of the non-competitive firms are close enough, capital accumulation increases in an egoistic steady state and decreases in an altruistic one. But, the impact of increasing productivity in the competitive sector can be reversed if the productivity of the less productive non-competitive firm is low enough.

JEL classification: D43, D9, O3

Keywords: imperfect competition, capital accumulation, technical progress.

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# 1 Introduction

This paper explores the consequences of imperfect competition on capital accumulation. We consider an economy populated by firms producing the same good and endowed with technologies that differ by their productivity. In a perfectly competitive economy, only the firm endowed with the most productive technology would be active. But the assumption of perfect competition becomes meaningless if only one firm (or a small number of firms) remains on the market. Therefore, the assumption of imperfect competition is natural for an economy populated by heterogeneous firms. We assume that the most productive firms realize that they hold some market power on the labor market, and take into account the impact of their demand for labor on wages. We thus obtain an equilibrium with strategic behavior in the labor market.

In an imperfect competition framework, technical progress affecting some firms may lead to new features. First, this higher productivity increases the monopsony power of the innovating firms with respect to other firms. By this effect, the distribution of income between labor and capital may change. Second, these changes in income distribution may lead to changes in capital accumulation. The direct effect of a technical progress is to increase capital accumulation. But, imperfect competition leads to indirect effects that may go in the same or in the opposite direction. Therefore, technical progress may have paradoxical effects on capital accumulation through imperfect competition.

The main assumptions of the model are the following. We assume that the single good of the economy is produced using capital and labor by different types of firms. First, competitive firms exist which all use the same basic Cobb-Douglas technology with constant returns. For a fixed (predetermined) level of capital, their competitive labor demand is a decreasing function of the real wage. Second, there exist  $m \geq 1$  non-competitive firms using their own technology that differs from the basic technology by having higher total factor productivity. This higher productivity can result from a past innovation process, and is exogenous. Moreover, we generally assume in this paper, for the sake of simplicity, that this production structure is identical in all periods. Non-competitive firms behave as an oligopsony on the labor market: they take into account the impact of their labor demand on the equilibrium wage and on their profits<sup>1</sup>. They play a Cournot-Nash game among themselves.

This production sector is part of a growth model with overlapping generations. The capital stock of each firm results from agents' investments in the preceding period. As agents can arbitrate between returns provided by different firms, all capital returns must be equal at equilibrium. This last condition added to equilibrium conditions of the Cournot-Walras game allow an equilibrium to be defined in which the shares of capital and labor used by each firm are endogenous.

We assume that agents are altruistic (as in Barro (1974) and Weil (1987)) and we focus on the long run steady state of the economy. It is well known that two

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<sup>1</sup>The article of Bhaskar, Manning and To (2002) gives many arguments in favor of the assumption of oligopsony competition in labor markets.

types of steady state may exist, depending on whether bequest is operative or not. Moreover, we assume a logarithmic instantaneous utility function. These assumptions have particular consequences on capital accumulation. In an egoistic steady state (without bequest), capital accumulation only depends on the equilibrium wage of the economy, as only labor earnings are saved. In contrast, in an altruistic steady state (with positive bequest), capital accumulation only depends on the rate of return of capital. The non-competitive behavior of some firms tends to decrease the equilibrium wage of the economy and to increase the rate of return of capital. Therefore, such behavior is detrimental to capital accumulation in an egoistic steady state and beneficial in an altruistic one.

In this framework, we study the impact on long run capital accumulation of an exogenous technical progress that affects one non-competitive firm, or the competitive sector. As a benchmark, it is worth noting that in a perfectly competitive economy, an increase in productivity always gives rise to an increase in capital accumulation.

Considering a non-competitive firm, it is proved that the impact of technical progress depends on the initial productivity of the firm and on the type of steady state (egoistic or altruistic). First, it should be noted that the increase in productivity of a non-competitive firm has opposing effects on factors remuneration at equilibrium: if the equilibrium wage increases, the rate of return decreases and vice versa. Therefore, if capital accumulation increases for an egoistic steady state, it will decrease for an altruistic one and vice versa. Second, an increase of one firm's productivity can have various consequences on factor remunerations, since it induces two effects. The direct effect is that the firm reduces the quantity of labor it uses per unit of capital. This effect tends to decrease the equilibrium value of the wage and to increase the capital return. The indirect effect is that the firm at equilibrium holds a higher share of the total capital of the economy. It acts in the same way as the first one, if the firm that benefits from the technical progress is initially the most productive one. But it acts in the opposing direction if the firm that benefits from this productivity increase is not initially the most productive one. The second effect may be dominant, particularly, for the less-productive non-competitive firm: an increase of its productivity can rise the equilibrium wage and decrease the capital return.

We consider now the impact on accumulation of an increase of productivity in the competitive sector. If the productivity levels of the non-competitive firms are close enough, we show that capital accumulation increases for an egoistic steady state and decreases for an altruistic one. This effect results from the increase of the wage and the decrease in the capital return. But, we show that the impact of increasing productivity in the competitive sector can be reversed if the productivity of the less-productive non-competitive firm is low enough.

From these results, it appears that technical progress may have paradoxical effects on capital accumulation through imperfect competition.

These results can be understood through comparison with existing literature. Indeed, our model can be viewed as the symmetrical to Sorger's (2002) and Becker's

(2003) contributions. In these articles, firms are perfectly competitive and consumers behave non-competitively in the capital market. In our model, consumers are perfectly competitive and some firms behave non-competitively in the labor market. In Sorger and Becker, a long run equilibrium exists in which consumers endowed with different rates of time preference hold positive amounts of capital. This result is in contrast with the competitive economy, in which only the most patient agent holds capital in the long run. In our contribution, a long run equilibrium exists in which firms endowed with different productivity levels are productive, while only the most productive ones would be active in a competitive setting. In both cases, the interpretation is the same: through his/her non-competitive behavior, an agent increases his/her gain, but it exerts a positive influence on the gain of other players. And agents who have the lowest market power are those who benefit the most from the non-competitive behavior of other agents. Therefore, all agents may remain in the market.

In Sorger and Becker, consumers endowed with the lowest rate of time preference have the greatest market power. They underinvest in capital in order to increase the rate of interest. But this behavior increases the gain of all other agents, and particularly the gain of agents who exercise "more competitive" behavior. In our framework, the most productive firms have stronger market power. They employ less labor for a given quantity of capital in order to diminish the equilibrium wage. But this behavior is beneficial to less-productive firms who employ more labor. Their lower technical progress is balanced by their higher demand of labor which increases their capital productivity.

The imperfect competition mechanism that we introduce in this paper can also be viewed in the line of Cournot-Walras equilibrium (cf. Gabszewicz and Vial (1972), Codognato and Gabszewicz (1993) and Gabszewicz and Michel (1997)). Following this concept, some agents, having a significant size compared to the whole economy, take into account the influence of their action on the equilibrium. A Walrasian equilibrium is formed which depends on the quantities chosen by the strategic agents who play a game of the Cournot-Nash type between themselves. Recent papers have used this concept in various frameworks. Belan, Michel and Wigniolle (2002) show that it can be fruitful to interpret pension funds behavior. Belan, Michel and Wigniolle (2005) wonder if imperfect competition can foster capital accumulation in a developing economy.

In the last part of the paper, a simple extension of the model is considered, which allows an evolving production structure to be studied. In each period, one firm may receive an innovation that increases its productivity by some given factor with respect to the common technology. These events occur at random and can be interpreted as the appearance of an innovation. The innovating firm has an exclusive use of its new technology during one period. In the next period, all firms benefit from the preceding innovation that becomes freely available. Therefore, there is one non-competitive firm in each period in which an innovation occurs, and there is no non-competitive firm in periods without innovation. In this framework, we study the dynamics of an egoistic equilibrium. We show that the occurrence of an innovation

has a negative short run impact on the growth rate, because imperfect competition decreases the equilibrium wage. But, after one period, the effect is positive as the technology is available for the competitive sector.

This simple model shows that a technical progress can have two paradoxical effects in the short run: first, it can cause a fall in capital accumulation; second, it can increase the share of GDP devoted to capital income and decrease the share devoted to labor income. These two effects result from the non-competitive behavior of innovating firms. Therefore, our model may provide an interpretation for two stylized facts that have been extensively discussed in recent literature. The first one is the Solow's paradox: from the eighties, numerous authors mention that innovations associated with computers and information technologies have not been associated with a significant jump in GDP growth rates (see for instance Solow (1987) and David (1990)). The second one is the fall of the share of GDP devoted to labor income and the jump of the share devoted to capital income that some Western countries have experienced from the seventies. For instance, in France between 1980 and 2000, the share of GDP devoted to labor income has experienced a fall from 72% to 60%<sup>2</sup> ((see for instance Askenazy (2003)). In our framework, these two stylized facts can be interpreted as the consequence of an increased monopsony power for innovating firms.

The paper is organized as follows. Section 2 presents the game played by non-competitive firms. Section 3 studies households' behavior. Section 4 presents the intertemporal long run equilibrium. Section 5 analyses the impact of an exogenous technical change on capital accumulation. Section 6 concludes. The most complex proofs are given in the appendix.

## 2 The productive sector

Two types of firms exist in the productive sector: competitive and non-competitive. We first introduce an equilibrium concept in which non-competitive firms behave as an oligopsony on the labor market: they take into account the impact of their labor demand on the equilibrium wage and on their profits. Second, the existence of an equilibrium of the game is proved. Third, the consequences of imperfect competition on equilibrium prices are studied.

### 2.1 Definition of the equilibrium concept

We consider an imperfect competition concept in the line of the Cournot-Walras equilibrium. At each period  $t$  occurs a game consisting of three steps. In a first step, households allocate their savings between the different firms, arbitrating between the different capital returns. At the second step, the non-competitive firms choose their labor demand (their strategic variable). In the third step, an equilibrium occurs on the labor market that determines the equilibrium wage.

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<sup>2</sup>During the same time, this share experiences oscillations in the US between 69 and 65 percent

We now make precise the assumptions regarding the productive sector.

Firms employ capital and labor. Capital depreciates fully in one period. There exist two types of firms: competitive and non-competitive. Competitive firms have the same Cobb-Douglas technology given by:

$$F(K_{0,t}, L_{0,t}) = A_0 K_{0,t}^\alpha L_{0,t}^{1-\alpha}$$

Without loss of generality, it is possible to consider only one competitive firm, and we denote by  $K_{0,t}$  and  $L_{0,t}$  its amounts of capital and labor.

There exist  $m$  non-competitive firms respectively endowed with Cobb-Douglas production technologies given by:

$$\text{for } i = 1, \dots, m, \quad A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha}$$

We assume the following inequalities:

$$A_0 < A_1 < \dots < A_m$$

Non-competitive firms have a higher total factor productivity than competitive firms. This higher productivity may result for instance from past innovations. We will assume along the first part of the paper that this production structure is identical in all periods.

At the beginning of period  $t$ , the total amount of capital  $K_t$  is allocated by households among the different firms. This capital stock results from period  $t - 1$  savings behavior and for the moment,  $K_t$  is assumed to be given<sup>3</sup>. In period  $t$ , the households who hold the capital stock of firm  $i$  ( $0 \leq i \leq m$ ) share the profit according to their capital contribution. The resulting payoff per unit of capital for them is:

$$R_{i,t} = \frac{A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha} - w_t L_{i,t}}{K_{i,t}} \quad (1)$$

We assume that households are atomistic and behave competitively. They take  $R_{i,t}$  as given and invest their savings in firms providing the highest returns  $R_{i,t}$ .

Using these assumptions, the three steps of the game are the following.

1. At the beginning of period  $t$ , consumers allocate their savings  $K_t$  among the different firms  $i$ ,  $0 \leq i \leq m$  :  $K_t = K_{0,t} + \sum_{i=1}^m K_{i,t}$ .
2. Their capital stock being installed, the  $m$  non-competitive firms choose their labor demand  $(L_{1,t}, \dots, L_{m,t})$ , such that

$$\sum_{i=1}^m L_{i,t} \leq N_t \quad (2)$$

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<sup>3</sup>Households' savings behavior is described in section 3.

3. There is an equilibrium on the labor market. This equilibrium is reached when the competitive labor demand of firm 0 is equal to the remaining quantity of labor after the decision of non-competitive firms, or

$$w_t = A_0(1 - \alpha)K_{0,t}^\alpha \left( N_t - \sum_{i=1}^m L_{i,t} \right)^{-\alpha}$$

As it is usual in Cournot-Walras competition, the strategies of non-competitive firms are constrained by (2) in such a way that an equilibrium exists. By assumption, firms  $i$  with  $1 \leq i \leq m$  are the only strategic agents. Firm 0 and consumers behave competitively.

The game is solved by backward induction.

From step 3, the equilibrium condition on the labor market defines the equilibrium wage as a function of the demand of labor by non-competitive firms  $L_{i,t}$ ,  $i = 1, \dots, m$ :

$$w_t = A_0(1 - \alpha)K_{0,t}^\alpha \left( N_t - \sum_{i=1}^m L_{i,t} \right)^{-\alpha} \equiv \omega_t(L_{1,t}, \dots, L_{m,t}) \quad (3)$$

From step 2, each non-competitive firm  $i$  maximizes its profit, taking into account the impact of its labor demand on the equilibrium wage:

$$\max_{L_{i,t}} A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha} - \omega_t(L_{1,t}, \dots, L_{m,t}) L_{i,t} \quad (4)$$

The optimal choice of  $L_{i,t}$  is such that<sup>4</sup>:

$$(1 - \alpha)A_i K_{i,t}^\alpha L_{i,t}^{-\alpha} - (1 - \alpha)A_0 K_{0,t}^\alpha L_{0,t}^{-\alpha} - (1 - \alpha)\alpha A_0 K_{0,t}^\alpha L_{i,t} L_{0,t}^{-\alpha-1} = 0 \quad (5)$$

The third term of this equation stems from the non-competitive behavior.

Finally, from step 1, all capital returns must be equal:

$$\forall i = 1, \dots, m, \frac{A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha} - w_t L_{i,t}}{K_{i,t}} = \frac{A_0 K_{0,t}^\alpha L_{0,t}^{1-\alpha} - w_t L_{0,t}}{K_{0,t}} = \alpha A_0 K_{0,t}^{\alpha-1} L_{0,t}^{1-\alpha} \quad (6)$$

and the total capital stock is shared between all firms:

$$K_{0,t} + \sum_{i=1}^m K_{i,t} = K_t \quad (7)$$

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<sup>4</sup>The concavity of the profit function with respect to  $L_{i,t}$  is satisfied, as the second derivative is negative:

$$-\alpha(1 - \alpha)A_i K_{i,t}^\alpha L_{i,t}^{-\alpha-1} - 2\alpha(1 - \alpha)A_0 K_{0,t}^\alpha L_{0,t}^{-\alpha-1} - (1 - \alpha)\alpha(1 + \alpha)A_0 K_{0,t}^\alpha L_{i,t} L_{0,t}^{-\alpha-2} > 0.$$



In order to characterize the equilibrium of the game, it is convenient to introduce the following notations:  $q_i = A_i/A_0$ ,  $l_{i,t} = L_{i,t}/K_{i,t}$ ,  $\lambda_{i,t} = l_{i,t}/l_{0,t}$ ,  $p_{i,t} = K_{i,t}/K_t$  and  $\tilde{p}_{i,t} = p_{i,t}/p_{0,t}$ .

Equation (5) can be written:

$$A_i l_{i,t}^{-\alpha} - A_0 l_{0,t}^{-\alpha} - \alpha A_0 \frac{K_{i,t}}{K_{0,t}} l_{i,t} l_{0,t}^{-\alpha-1} = 0$$

Dividing by  $A_0 l_{0,t}^{-\alpha}$  we obtain:

$$q_i \lambda_{i,t}^{-\alpha} - 1 - \alpha \tilde{p}_{i,t} \lambda_{i,t} = 0 \quad (8)$$

The equality of capital returns for each firm (6) defines the gross return on savings  $R_t$ :

$$R_t \equiv \alpha A_0 l_{0,t}^{1-\alpha} = A_i l_{i,t}^{1-\alpha} - A_0 (1-\alpha) l_{0,t}^{-\alpha} l_{i,t} \quad (9)$$

Dividing by  $A_0 l_{0,t}^{-\alpha}$  we obtain the equation:

$$q_i \lambda_{i,t}^{1-\alpha} - (1-\alpha) \lambda_{i,t} = \alpha \quad (10)$$

Finally, the allocation of total capital on the different firms (7) leads to:

$$p_{0,t} + \sum_{i=1}^m p_{i,t} = 1$$

or:

$$p_{0,t} \left( 1 + \sum_{i=1}^m \tilde{p}_{i,t} \right) = 1 \quad (11)$$

Equations (8), (10) and (11) allow to characterize the equilibrium of the game.

## 2.2 Existence of the equilibrium between non-competitive firms

In this section, we prove that our equilibrium concept leads to a unique equilibrium, and we describe its properties. We first remark that (8), (10) and (11) define a system of  $2m+1$  equations for  $2m+1$  variables, and that these equations do not depend on the period  $t$ . Consequently,  $\forall i = 1, \dots, m$ ,  $\lambda_{i,t}$ ,  $\tilde{p}_{i,t}$  and  $p_{0,t}$  are constant variables, that we will write further  $\lambda_i$ ,  $\tilde{p}_i$  and  $p_0$ .

From equation (8), we obtain that  $\lambda_i$  is such that:  $q_i \lambda_i^{-\alpha} > 1$ , or  $\lambda_i < (q_i)^{1/\alpha}$ . Equation (10) has a unique solution  $\lambda_i$  such that  $\lambda_i < (q_i)^{1/\alpha}$ , and this solution defines  $\lambda_i$  as a decreasing function of  $q_i$ . Moreover, as  $q_i \lambda_i^{-\alpha} > 1$ , (10) implies that:

$$\lambda_i - (1-\alpha) \lambda_i - \alpha < \lambda_i q_i \lambda_i^{-\alpha} - (1-\alpha) \lambda_i - \alpha = 0$$

Thus,

$$\lambda_i < 1$$

We have finally proved that all non-competitive firms have a smaller labor-capital ratio than the ratio in the competitive sector. The more productive a non-competitive firm is, the smaller its labor-capital ratio is, as  $\lambda_i$  is a decreasing function of  $q_i$ . This property results from the higher market power of the more productive firms: they reduce their labor demand in order to decrease the equilibrium wage.

As for all  $i = 1, \dots, m$ ,  $\lambda_i$  is well-defined, we deduce from (8) the value of  $\tilde{p}_i$ ,  $\forall i = 1, \dots, m$  :

$$\tilde{p}_i = \frac{q_i \lambda_i^{-\alpha} - 1}{\alpha \lambda_i}$$

which is an increasing function of  $q_i$ .

Finally  $p_0$  is given by (11), and is a decreasing function of  $q_i$ . The share of capital held by firm  $i$   $p_i$  is given by:

$$p_i = p_0 \tilde{p}_i = \frac{\tilde{p}_i}{1 + \sum_{j=1}^m \tilde{p}_j}$$

which is an increasing function of  $\tilde{p}_i$ , and therefore an increasing function of  $q_i$ .

We have finally proved that, for each value of total capital, there exists a non-competitive equilibrium in which all firms are productive. The higher the productivity of a firm is, the higher the share of capital that it employs at equilibrium is and the lower its labor-capital ratio will be. It is worth noting that, in an equilibrium with perfect competition, only the most productive firm would be active. With imperfect competition, the more productive firms strategically diminishes their labor demand to decrease the equilibrium wage. This behavior exerts a positive externality on less productive firms, which employ a higher labor-capital ratio, and which can attain the same level of capital productivity.

## 2.3 Equilibrium prices

At period  $t$ ,  $K_t$  being the total capital stock and  $N_t$  the number of young people, it is possible to determine the equilibrium level of the wage  $w_t$  and the gross interest rate  $R_t$ .

From the labor market equilibrium, we have:

$$\begin{aligned} N_t &= \sum_{i=0}^m L_{i,t} = \sum_{i=0}^m l_{i,t} p_i K_t \\ N_t &= l_{0,t} p_0 K_t \left[ 1 + \sum_{i=1}^m \lambda_i \tilde{p}_i \right] \end{aligned}$$

Finally, with  $k_t = K_t/N_t$  denoting the ratio of capital per young agent, we obtain from (11):

$$\begin{aligned} l_{0,t}^{-1} &= k_t X \\ \text{with } X &= \frac{1 + \sum_{i=1}^m \lambda_i \tilde{p}_i}{1 + \sum_{i=1}^m \tilde{p}_i} \end{aligned}$$

In an economy with perfect competition, we would obtain  $X = 1$ . In our economy with imperfect competition, we have  $X < 1$  as  $\lambda_i < 1$ . For the competitive firm, the equilibrium labor-capital ratio  $l_{0,t}$  is higher than its value for a perfectly competitive economy  $1/k_t$ .

We can then deduce the values of the equilibrium wage (3) and gross interest rate (9):

$$w_t = (1 - \alpha)A_0 l_{0,t}^{-\alpha} = (1 - \alpha)A_0 k_t^\alpha X^\alpha \quad (12)$$

$$R_t = \alpha A_0 l_{0,t}^{1-\alpha} = \alpha A_0 k_t^{\alpha-1} X^{\alpha-1} \quad (13)$$

In these two equations, the variable  $X$  results from imperfect competition. As  $X < 1$ , we see that imperfect competition tends to decrease the equilibrium wage, and to increase the gross interest rate, with respect to the case of perfect competition with the less productive technology.

### 3 Households' behavior

The production sector is part of a growth model with overlapping generations of altruistic agents. The model is based on Diamond (1965) and Barro (1974)-Weil (1987). Agents are living for two periods. The size of generation  $t$  is  $N_t$  and each agent has  $(1 + n)$  children. We assume that parents care about their children's welfare by weighting their children's utility in their own utility function. The utility of a generation born at time  $t$ ,  $V_t$ , is given by

$$V_t = U(c_t, d_{t+1}) + \gamma V_{t+1}, \quad 0 < \gamma < 1$$

with  $U(c_t, d_{t+1}) = (1 - a) \ln(c_t) + a \ln(d_{t+1})$

$c_t$  and  $d_{t+1}$  respectively denote first period and second period consumptions. In their first period of life, individuals born in  $t$  work and receive a wage  $w_t$ . In addition to their wage income, they receive a bequest  $x_t$  from their parents. They consume  $c_t$  and save an amount  $s_t$ . Gross returns on savings are equal to  $R_{t+1}$ : at equilibrium, all firms provide the same return on capital. In their second period of life, people receive returns on savings and allocate net resources between consumption  $d_{t+1}$  and bequests  $x_{t+1}$  to their  $(1 + n)$  children. Thus

$$x_t + w_t = c_t + s_t \quad (14)$$

$$R_{t+1} s_t = d_{t+1} + (1 + n) x_{t+1} \quad (15)$$

Bequests must be non-negative :

$$x_{t+1} \geq 0 \quad (16)$$

The maximum of total utility is given by the following recursive relation:

$$V_t^*(x_t) = \max_{c_t, s_t, d_{t+1}, x_{t+1}} \{U(c_t, d_{t+1}) + \gamma V_{t+1}^*(x_{t+1})\}$$

subject to (14), (15) and (16).

For any positive  $t$ ,  $V_t^*(x_t)$  represents the maximum utility of a young agent born in  $t$  when he inherits  $x_t$ . These are the value functions of the infinite horizon problems  $\max \sum_{j=0}^{+\infty} \gamma^j U(c_{t+j}, d_{t+j+1})$  subject to (14), (15) and (16).

This maximization problem leads to the following first-order conditions

$$U'_c(c_t, d_{t+1}) = R_{t+1} U'_d(c_t, d_{t+1}) \quad (17)$$

$$-(1+n)U'_d(c_t, d_{t+1}) + \gamma U'_c(c_{t+1}, d_{t+2}) \leq 0 \quad (18)$$

The second condition holds with equality if  $x_{t+1} > 0$ . Equation (17) is the standard condition for individual life-cycle allocation. Condition (18) is a condition for optimal allocation between parent and children. If  $x_{t+1} > 0$ , it states that the marginal utility loss from reduction of a parent's consumption will equal the marginal utility gain of an increase in the bequest.

When bequests are constrained at all periods ( $\forall t, x_t = 0$ ), equation (17) with a log-linear utility function leads to the simple saving function:

$$s_t = aw_t \quad (19)$$

In the long run, the economy reaches a steady state that is called egoistic long run equilibrium.

When bequests are positive, it is well known that the economy converges towards the modified golden rule steady state:

$$R = \frac{1+n}{\gamma} \quad (20)$$

We call altruistic long run equilibrium this steady state.

The equilibrium values of prices  $w$  and  $R$  will depend on the equilibrium between non-competitive firms.

## 4 The intertemporal long run equilibrium

Two types of long run intertemporal equilibria may exist: an altruistic equilibrium with operative bequest ( $x > 0$ ) and an egoistic equilibrium ( $x = 0$ ).

At an altruistic long run equilibrium, the capital per young agent ratio  $k_t$  converges towards a value  $\bar{k}$ , which is determined by (13) and by the modified golden rule (20):

$$R = \alpha A_0 \bar{k}^{\alpha-1} X^{\alpha-1} = \frac{1+n}{\gamma}$$

or:

$$\bar{k} = \left( \frac{\alpha\gamma}{1+n} \right)^{\frac{1}{1-\alpha}} \frac{(A_0)^{\frac{1}{1-\alpha}}}{X} \quad (21)$$

Along an egoistic long run equilibrium, the capital per young agent ratio  $k_t$  converges toward a value  $k^*$ , which is determined by (12) and by the savings behavior of the agents (19):

$$(1+n)k^* = aw$$

or:

$$k^* = \left( \frac{(1-\alpha)a}{1+n} \right)^{\frac{1}{1-\alpha}} (A_0)^{\frac{1}{1-\alpha}} (X)^{\frac{\alpha}{1-\alpha}} \quad (22)$$

In both equations (21) and (22) the impact of imperfect competition on capital accumulation results from the variable  $X < 1$ . Within an altruistic steady state, capital accumulation in the long run only depends on the return of capital. As imperfect competition tends to increase the gross interest rate, the capital per young agent ratio  $\bar{k}$  is higher than under perfect competition. In contrast, within an egoistic steady state, capital accumulation in the long run only depends on savings that only depend on the equilibrium wage for a log-linear utility function. Thus, as imperfect competition tends to decrease the equilibrium wage, the capital per young agent ratio  $k^*$  is smaller than under perfect competition.

It is straightforward to show that the condition ensuring positive bequests in Weil (1987) remains the same in our framework. The steady state will be altruistic if  $\bar{k} > k^*$ . This inequality gives the following condition:

$$(1-\alpha)aX < \alpha\gamma$$

As  $X$  is smaller than 1, this condition shows that the existence of an altruistic steady state is furthered by imperfect competition. This property was expected as imperfect competition tends to increase the return to capital.

## 5 Technical progress and capital accumulation

In this section, we study how technical progress (i.e. an increase of some  $A_i$ ,  $i = 0, \dots, m$ ) modifies capital accumulation in the long run.

### 5.1 Increasing productivity of a non-competitive firm

We first study the impact of an increase of some  $A_i$ ,  $i \geq 1$ . This is equivalent to consider that some  $q_i$  increases, for  $i \geq 1$ . Such technical progress will affect capital accumulation through the variable  $X$ . It is worth noting that  $X$  has an opposite effect on the two types of steady states, altruistic or egoistic. An increase of  $X$  diminishes  $\bar{k}$  and rises  $k^*$ .

As a benchmark, we know that a technical progress in a competitive economy always increases capital accumulation, in both types of long run equilibria.

**Proposition 1** *it is possible to define some increasing function  $\kappa(q)$  with  $\kappa(1) = 0$ , such that, for each  $q_i$ ,  $i = 1, \dots, m$ , the interval  $K_i = (1, q_i + \kappa(q_i))$  satisfies:*

1. *If for all  $j \neq i$ ,  $q_j \in K_i$ ,  $\frac{\partial X}{\partial q_i} < 0$ . An increase of  $q_i$  rises  $\bar{k}$  and diminishes  $k^*$ .*
2. *If for all  $j \neq i$ ,  $q_j \notin K_i$ ,  $\frac{\partial X}{\partial q_i} > 0$ . An increase of  $q_i$  diminishes  $\bar{k}$  and raises  $k^*$ .*

- Corollary 1**    1. For  $i = m$ , as all  $q_j \in K_m$ , we have  $\frac{\partial X}{\partial q_m} < 0$ . An increase of  $q_m$  raises  $\bar{k}$  and diminishes  $k^*$ .
2.  $q_1$  is the only variable that can satisfy point 2 of proposition (1). Particularly, if  $q_1$  is sufficiently small, ( $q_1$  tends toward 1),  $\frac{\partial X}{\partial q_1} > 0$ . An increase of  $q_1$  diminishes  $\bar{k}$  and raises  $k^*$ .

These results show that a technical progress can have various effects on capital accumulation. The first case is obtained when all firms  $j$  have a productivity parameter  $q_j$  close to  $q_i$  (close in the sense that  $q_j \in (1, q_i + \kappa(q_i))$ ). The second case is obtained when  $q_1$  is sufficiently small with respect to  $q_2, q_3, \dots, q_m$ . Therefore, the impact of a technical progress on capital accumulation depends on two components: on the initial productivity of the firm experiencing a technical progress with respect to other firms, and on the type of long-run equilibrium - egoistic or altruistic.

This latter component can be understood, having in mind that the rise of productivity of a non-competitive firm has opposite effects on factors remuneration at equilibrium: if the equilibrium wage increases, the rate of return decreases and vice versa. Therefore, if capital accumulation increases for an egoistic steady state, it will decrease for an altruistic one and conversely.

The former component can be explained, as an increase of one firm's productivity leads to two opposite effects. First, the firm reduces the quantity of labor used per unit of capital. This effect tends to decrease the equilibrium value of the wage and to increase the capital return. Second, since all capital returns are equal at equilibrium, the firm holds a higher share of the total capital of the economy. If the firm that benefits from this productivity increase is the most productive ( $i = m$ ), this second effect acts in the same sense as the first one. Therefore, the equilibrium wage decreases and the capital return increases, which decreases accumulation for an egoistic steady state and increases accumulation for an altruistic one. But, if the firm that benefits from this productivity increase is not the most productive one ( $i < m$ ), the second effect acts in opposite direction to the first one, and the global effect is ambiguous. Particularly, it is proved that for the less productive non-competitive firm, it is possible that a rise in its productivity increases the equilibrium wage and decreases the capital return.

## 5.2 Increasing productivity in the competitive sector

We study the impact of an increase of  $A_0$  on both types of stationary equilibrium  $\bar{k}$  and  $k^*$ . From (21) and (22),  $A_0$  has a direct effect and an indirect effect via the variable  $X$ . Indeed, increasing  $A_0$  implies a decrease for all  $q_i$ ,  $i \geq 1$ .

The following proposition shows that an increase of  $A_0$  can have various effects on capital accumulation.

- Proposition 2**    1. When  $q_m \rightarrow +\infty$ ,  $\frac{d \ln k^*}{d A_0} \rightarrow \frac{1}{A_0(1-\alpha)}$  and  $\frac{d \ln \bar{k}}{d A_0} \rightarrow 0$ .
2. When  $q_1 \rightarrow 1$ ,  $\frac{d \ln k^*}{d A_0} \rightarrow -\infty$  and  $\frac{d \ln \bar{k}}{d A_0} \rightarrow +\infty$ .

3. When  $\forall i \geq 1$ ,  $q_i = q$ ,  $\frac{d \ln k^*}{d A_0} > 0$  and  $\frac{d \ln \bar{k}}{d A_0} < 0$ . By continuity, these properties hold when the  $q_i$ ,  $i \geq 1$ , are sufficiently close together.

These results show that when the total factor productivities of non-competitive firms are close, an increase of  $A_0$  tends to increase capital accumulation in an egoistic steady state, and to decrease capital accumulation in an altruistic steady state. This effect results from the increase of the wage and the decrease of the capital return. In contrast, the results may be reversed when the total factor productivities of non-competitive firms are distant.

These properties can be interpreted as follows. The direct effect of an increase of the productivity in the non-competitive sector is a rise of both wage and capital returns. But an indirect effect stems from the fall of the relative productivity of all non-competitive firms, which modify their market power. If non-competitive firms have close productivities (case 3), the productivity increase in the competitive sector implies a fall in the market power of all non-competitive firms, which causes an increase of wages and a decrease of capital returns. If non-competitive firms have distant productivities (cases 1 and 2), the productivity increase in the competitive sector redistributes market power in favor of the most productive firms, and to the detriment of the less productive ones. The resulting effect on capital accumulation may be reversed with respect to case 3.

### 5.3 Growth with random innovations

In this last section, we provide a simple extension of the model, introducing random innovations. We assume that at each period, with a probability  $\pi$ , one firm receives an innovation (and with probability  $1 - \pi$ , no innovation occurs in the whole economy). This innovation increases the productivity by a factor  $\delta > 1$ , with respect to the common technology. Finally, each innovator has an exclusive use of its new technology during only one period. After that period, there is free access to this technology.

From the preceding assumptions, at each period  $t$ , either the economy is purely competitive and the common productivity is  $A_{0,t}$ , or one non-competitive firm has a productivity level  $A_{1,t} = \delta A_{0,t}$  while the other firms are competitive with the common productivity  $A_{0,t}$ .

We only consider the egoistic equilibrium in this part, as we want to analyze the dynamics of capital accumulation. We use the expression of  $X$  given by equation (23), that has been proved in appendix 1, setting  $x = 1/\lambda$ . The dynamics of  $k_t$  with random innovations is:

$$(1 + n)k_{t+1} = a(1 - \alpha)A_{0,t}k_t^\alpha X_t^\alpha$$

With probability  $\pi$  (arrival of an innovation in  $t$ )

$$\begin{aligned} X_t &= \frac{x}{1 + x(x - 1)} \\ \text{with } x &> 1 \text{ the solution of } \delta = \alpha x^{1-\alpha} + (1 - \alpha)x^{-\alpha} \\ A_{0,t+1} &= \delta A_{0,t} \end{aligned}$$

With probability  $1 - \pi$  (no innovation in  $t$ )

$$\begin{aligned} X_t &= 1 \\ A_{0,t+1} &= A_{0,t} \end{aligned}$$

As  $x > 1$ , we have  $\frac{x}{1+x(x-1)} < 1$ . The arrival of an innovation has a negative short run effect on the growth rate, because imperfect competition decreases the equilibrium wage. But, after one period, the effect is positive as the technology is available for the competitive sector. The greater the size of the innovation  $\delta$  is, the larger both short run and long run effects will be.

This simple example shows that a technical progress can have a negative short run impact on growth, associated with a fall of wages and an increase of capital returns. All these facts results from the increase in the monopsony power of the innovating firm. This model could provide an interpretation for some stylized facts that have been extensively discussed in recent literature. More precisely, two stylized facts could be interpreted (partially) with our model. First, from the eighties, numerous authors mention that innovations associated with computers and information technologies have not been associated with a significant jump in GDP growth rates, or that their impact on productivity has been delayed. This idea is become very popular under the name of Solow's paradox (see Solow (1987) and David (1990)). Second, from the seventies, some western countries have experienced a fall of the share of GDP devoted to labor income and a jump of the share devoted to capital income. In our framework, these two stylized facts can be interpreted as the consequence of an increased monopsony power for innovating firms.

## 6 Conclusion

This paper has studied how long run growth can be affected by strategic behavior of firms in the labor market. The main results show the paradoxical effects associated with imperfect competition: a technical progress may decrease capital accumulation if it leads to distortions due to imperfect competition.

Our work could lead to further developments, mostly in endogenizing the technical progress by an explicit innovative activity of the firms. Growth models in which innovation is the source of growth are natural frameworks to develop our analysis, since they make endogenous productivity differences of firms.

### Appendix 1: proof of proposition 1

We study the impact of an increase of  $q_i$  on  $X$ , defined by

$$X = \frac{1 + \sum_{j=1}^m \lambda_j \tilde{p}_j}{1 + \sum_{j=1}^m \tilde{p}_j}$$

From (8), we have:

$$\tilde{p}_j = \frac{q_j \lambda_j^{-\alpha} - 1}{\alpha \lambda_j}$$



From (10), we have:

$$q_j \lambda_j^{-\alpha} = (1 - \alpha) + \frac{\alpha}{\lambda_j}$$

Thus, we obtain,

$$\tilde{p}_j = \frac{1}{\lambda_j} \left( \frac{1}{\lambda_j} - 1 \right)$$

We define for each  $j$ ,  $x_j = 1/\lambda_j$ . It is then possible to write

$$X = \frac{1 + \sum_{j=1}^m (x_j - 1)}{1 + \sum_{j=1}^m x_j (x_j - 1)} \quad (23)$$

By definition of  $x_j$  and from equation (10),  $x_j$  is the solution greater than 1 of the equation:

$$q_i = \alpha x_i^{1-\alpha} + (1 - \alpha) x_i^{-\alpha}$$

Therefore,  $x_j$  increases with  $q_j$  and we have  $1 < x_1 < \dots < x_m$ .

Computing the derivation  $\frac{\partial X}{\partial x_i}$ , its sign is the same as the sign of  $Z$  such that:

$$Z = 1 + \sum_{j=1}^m x_j (x_j - 1) - \left[ 1 + \sum_j (x_j - 1) \right] (2x_i - 1)$$

We take as given the value of  $x_i$ , and we study  $Z$  as a function of  $x_j$ ,  $j \neq i$ . It is possible to write:

$$Z = 1 - x_i^2 + \sum_{j \neq i} P(x_j, x_i)$$

with  $P(x_j, x_i)$  such that:

$$P(x_j, x_i) = x_j^2 - x_j - (2x_i - 1)(x_j - 1) = (x_j - 1)(x_j - 2x_i + 1)$$

For a given  $x_i$ , we study  $P(x_j, x_i)$  as a function of  $x_j$ . This function is quadratic and reaches its minimum in  $x_j$  solution of  $\partial P(x_j, x_i)/\partial x_j = 0$ , which gives

$$2x_j - 1 - (2x_i - 1) = 0$$

or:

$$x_j = x_i$$

Thus  $Z$  reaches a minimum for  $x_j = x_i \forall j \neq i$ , and the minimum value  $Z_{\min}$  is:

$$Z_{\min} = 1 - x_i^2 - (m - 1)(x_i - 1)^2 = -(x_i - 1) [m(x_i - 1) + 2] < 0$$

As all functions  $x_j \mapsto P(x_j, x_i)$  are identical and symmetrical with respect to the minimum reached in  $x_j = x_i$ , there exists an interval  $I_i = (x_i - \rho(x_i), x_i + \rho(x_i))$ ,

such that, if all  $x_j$  are in  $I_i$ , then  $Z < 0$ , and if all  $x_j \notin I_i$ , then  $Z > 0$ .  $\rho(x_i)$  is defined by the property: if  $\forall j \neq i, x_j = x_i \pm \rho(x_i)$ ,  $X = 0$ .

But, it is easy to see that the lower bound of the interval  $x_i - \rho(x_i)$  is smaller than 1. Indeed, for  $x_j = 1 \ \forall j \neq i$ , we have  $X = 1 - x_i^2 < 0$ , which means that  $x_i - \rho(x_i) < 1$ . Thus, the relevant interval is in fact:  $J_i = (1, x_i + \rho(x_i))$ .

Finally, the value of  $\rho(x_i)$  is computed as the value of  $\rho$  solution of the equation:  $X = 0$  with  $\forall j \neq i, x_j = x_i + \rho$ . This leads to the equation:

$$\rho^2 = \frac{mx_i^2 - 2(m-1)x_i + m - 2}{m-1}$$

which defines  $\rho$  as a function of  $x_i$ :

$$\rho(x_i) = \sqrt{\frac{mx_i^2 - 2(m-1)x_i + m - 2}{m-1}}$$

The interval  $J_i = (1, x_i + \rho(x_i))$  implicitly defines an interval  $K_i$  on  $q_i$  under the form  $(1, q_i + \kappa(q_i))$ . Indeed, equation (10) implicitly defines  $x_i$  as a function of  $q_i$ :  $x_i$  is the solution  $> 1$  of the equation:

$$q_i = \alpha x_i^{1-\alpha} + (1-\alpha)x_i^{-\alpha}$$

We denote by  $\chi$  this function.  $x_i = \chi(q_i)$  is an increasing (bijective) function from  $[1, +\infty)$  on to  $[1, +\infty)$ .

We have:

$$\frac{\partial X}{\partial q_i} = \frac{\partial X}{\partial x_i} \frac{dx_i}{dq_i}$$

with

$$\frac{dx_i}{dq_i} = \chi'(x_i) = \frac{1}{\alpha(1-\alpha)x_i^{-1-\alpha}(x_i-1)} > 0$$

Therefore  $\frac{\partial X}{\partial q_i}$  and  $\frac{\partial X}{\partial x_i}$  have the same sign.

Finally, we obtain the interval  $K_i = (1, q_i + \kappa(q_i))$  such that, for all  $q_j$  in this interval,  $\frac{\partial X}{\partial q_i} < 0$ . The function  $\kappa(q_i)$  is defined as:

$$\begin{aligned} q_i + \kappa(q_i) &= \chi^{-1}(x_i) \\ \text{with } x_i &= \chi(q_i) \end{aligned}$$

or:

$$q_i + \kappa(q_i) = \alpha (\chi(q_i) + \rho(\chi(q_i)))^{1-\alpha} + (1-\alpha) (\chi(q_i) + \rho(\chi(q_i)))^{-\alpha}$$

## Appendix 2: proof of proposition 2

We first compute the expression of  $\frac{dx_i}{dA_0}$  for all  $i$ .

As  $q_i = A_i/A_0$ , we have:  $\frac{dq_i}{dA_0} = -\frac{q_i}{A_0}$ . From equation (10),  $x_i$  is defined as a function of  $q_i$ , and we have:

$$\frac{dx_i}{dq_i} = \frac{1}{\alpha(1-\alpha)x_i^{-1-\alpha}(x_i-1)}$$

Therefore,

$$\begin{aligned}\frac{dx_i}{dA_0} &= \frac{dx_i}{dq_i} \frac{dq_i}{dA_0} \\ &= -\frac{q_i}{A_0} \frac{1}{\alpha(1-\alpha)x_i^{-1-\alpha}(x_i-1)}\end{aligned}$$

and using equation (10) to eliminate  $q_i$ , we obtain:

$$\frac{dx_i}{dA_0} = \frac{-1}{\alpha(1-\alpha)A_0} \frac{x_i}{x_i-1} (\alpha x_i + 1 - \alpha)$$

Second, we compute the expression of  $\frac{d \ln X}{dA_0}$ , with  $X$  defined as a function of  $(x_1, \dots, x_m)$ :

$$X = \frac{1 + \sum_{j=1}^m (x_j - 1)}{1 + \sum_{j=1}^m x_j (x_j - 1)}$$

We obtain:

$$\frac{d \ln X}{dA_0} = \frac{1}{\alpha(1-\alpha)A_0} \sum_{i=1}^m \frac{x_i(\alpha x_i + 1 - \alpha)}{x_i - 1} \frac{\left[ (2x_i - 1) \left( 1 + \sum_{j=1}^m (x_j - 1) \right) - \left( 1 + \sum_{j=1}^m x_j (x_j - 1) \right) \right]}{\left( 1 + \sum_{j=1}^m x_j (x_j - 1) \right) \left( 1 + \sum_{j=1}^m (x_j - 1) \right)}$$

From this formula, we have:

$$\lim_{x_m \rightarrow +\infty} \frac{d \ln X}{dA_0} = \frac{1}{(1-\alpha)A_0}$$

$$\lim_{x_1 \rightarrow 1} \frac{d \ln X}{dA_0} = -\infty$$

And, if we take  $x_i = x \ \forall i$ , we obtain:

$$\frac{d \ln X}{dA_0} = \frac{1}{\alpha(1-\alpha)A_0} \frac{mx(\alpha x + 1 - \alpha) [m(x-1) + 2]}{[1 + mx(x-1)] [1 + m(x-1)]} \quad (24)$$

Finally we have to consider the two stationary states.

For an altruistic steady state, the sign of  $\frac{d \ln \bar{k}}{dA_0}$  is given, from (21), by the sign of

$$\frac{1}{(1-\alpha)A_0} - \frac{d \ln X}{dA_0}$$

For an egoistic steady state, the sign of  $\frac{d \ln k^*}{d A_0}$  is given, from (22), by the sign of

$$\frac{1}{A_0} + \alpha \frac{d \ln X}{d A_0}$$

Items 1 and 2 of proposition 2 immediately follow from the preceding results.

The result  $\frac{d \ln k^*}{d A_0} > 0$  of point 3 is obtained as  $\frac{d \ln X}{d A_0} > 0$  from (24). Finally,  $\frac{d \ln \bar{k}}{d A_0} < 0$  is obtained in using (24) to calculate:

$$\begin{aligned} \frac{1}{(1-\alpha)A_0} - \frac{d \ln X}{d A_0} &= \frac{1}{\alpha(1-\alpha)A_0} \left\{ \alpha - \frac{mx(\alpha x + 1 - \alpha) [m(x-1) + 2]}{[1 + mx(x-1)] [1 + m(x-1)]} \right\} \\ &= \frac{\frac{m^2}{\alpha} x(1-x) - mx^2 - 2mx \frac{1-\alpha}{\alpha} - m + 1}{\alpha(1-\alpha)A_0 [1 + mx(x-1)] [1 + m(x-1)]} \end{aligned}$$

In the last expression, the denominator is positive and the numerator is always negative for  $x > 1$  and  $m \geq 1$ .

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